B.Sc. Part II Diff. eqns

LDE CC

I. Solve $(D^2+1)y = x^2 \sin 2x$.

Soln For CF $D^2+1=0 \Rightarrow D = \pm i$

$\therefore CF = c_1 \cos x + c_2 \sin x$ —(1)

For PI $y = \dfrac{1}{D^2+1} x^2 \sin 2x$

$\Rightarrow y = \text{Imaginary part of } \dfrac{1}{D^2+1} x^2(\cos 2x + i \sin 2x)$

$= \text{I.P. of } \dfrac{1}{D^2+1} x^2 e^{2ix}$

$= \text{IP of } e^{2ix} \dfrac{1}{(D+2i)^2+1} x^2$

$= \text{IP of } e^{2ix} \dfrac{1}{D^2+4Di-3} x^2$

$= \text{I.P of } \dfrac{e^{2ix}}{-3} \cdot \dfrac{1}{\left[1 - \dfrac{D^2+4Di}{3}\right]} x^2$

$= \text{IP of } \dfrac{e^{2ix}}{-3} \left[1 - \dfrac{D^2+4Di}{3}\right]^{-1} x^2$

or, $$y = \text{I.P. of } \frac{e^{2ix}}{-3}\left[1+\left(\frac{D^2+4Di}{3}\right)+\left(\frac{D^2+4Di}{3}\right)^2 + \cdots\right]x^2$$

$$\Rightarrow y = \text{IP of } \frac{e^{2ix}}{-3}\left[1+\left(\frac{D^2+4Di}{3}\right)x^2+\left(\frac{D^2+4Di}{3}\right)^2 x^2 + \text{higher powers of } D\right]$$

$$\Rightarrow y = \text{IP of } \frac{e^{2ix}}{-3}\left[x^2 + \frac{D^2(x^2)+4iD(x^2)}{3} + \frac{D^4(x^2)+16i^2D^2(x^2)+8iD^3(x^2)}{9} + \text{higher powers of } D\right]$$

$$\Rightarrow y = \text{IP of } \frac{e^{2ix}}{-3}\left[x^2 + \frac{2+4i\times 2x}{3} + \frac{0-16\times 2+0}{9}+0\right]$$

$$\Rightarrow y = \text{IP of } \frac{e^{2ix}}{-3}\left(x^2+\frac{2+8xi}{3}-\frac{32}{9}\right)$$

$$= \text{IP of } \frac{e^{2ix}}{-3}\left(x^2+\frac{8xi}{3}-\frac{26}{9}\right)$$

$$\Rightarrow PI = \frac{8x\cos 2x}{-9} - \frac{\sin 2x}{3}\left(x^2-\frac{26}{9}\right) \quad \text{---(2)}$$

Hence, the complete soln is $y = CF + PI$, given by (1) & (2)